Sacred Heart Senior Secondary School, Dalhousie
Pre- Board, Subject- Mathematics

Class-12th M.M-80

Date: 12-01-26 TIME-3hrs

Q1. [15×1]

(1) The number of all possible matrices of order 3x3 with each entry 0 or 1 is:

(a) 27 (b) 8 (c) 81 (d) 512.

(2) Principal value of sin-1(sin 2π/3) is:

(a) -2π/3 (b)2 π/3 (c) π/3 (d) none of these

(3) Matrices A and B will be inverse of each other only if:

(a) AB = BA (b) AB-BA = 0 (c) AB = 0, BA = I (d) AB = BA = I

(4) If A,B are symmetric matrices of same order,then AB-BA is a

(a) skew symmetric matrix

(b) symmetric matrix

(c) zero matrix

(d) identity matrix.

(5) If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then

(a) P(B|A) = 1 (b) P(A|B) = 1 (c) P(B|A) = 0 (d) P(A|B) = 0

(6) Which of the following is a homogeneous differential equation?

(a) (4x + 6y + 5) dy – (3y + 2x + 4) dx = 0

(b) (xy) dx – (x^3+y^3) dy = 0

(c) (x^3 + 2y^2) dx + 2xy dy = 0

(d) y^2 dx + (x^2 – xy –y^2) dy = 0

(7) If A is a 2 x 2 matrix with |A| = 5 then the value of |A^T| will be :

(a) 5 (b) 1/5 (c) 25 (d) 125

(8) Derivative of $(tan^{-1}x + cot^{-1}x) =$

(a) 0 (b) sinx+cosx (c) 1 (d) none of these.

(9) The function f(x)=3x+17 is:

(a) strictly increasing on R

(b) strictly decreasing on R

(c) neither increasing nor decreasing on R

(d) none of these

(10) The radius of a circle is increasing at the rate of 0.7cm/s. Then the rate of increase of its circumference is:

(a) 1.4πcm/s (b) 2.4πcm/s (c) 0.4cm/s (d) -0.4cm/s.

(11) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0$ is :

(a) 3 (b) 2 (c) 1 (d) not defined.

(12) The line y = mx + 1 is a tangent to the curve y2 = 4x if the value of m is

(a) 1 (b) 2 (c) 3 (d)1/2

(13) If a is a unit vector and (x+a).(x-a)=8. Find $|x|$

(14) If events A and B are independent, such that $\mathrm{P(A)} = \frac{4}{7}$, $P(B) = \frac{14}{5}$, find $P(A \cup B)$.

(15) Using L'Hospital's Rule, evaluate: $\lim_{x\to 2} \frac{x^3-7x^2+10x}{x^2-x+6}$.

Q2. Let A = R – {5} and B = R – {1}. If f : A → B is a function defined by f(x) = $\frac{x-3}{x-5}$ then show that f is a one-one and an onto function. [2]

Q3. Find the value of p if A and B are independent events such that P(A) = 0.4, P(B) = p and P(A∪B) = 0.6 [2]

Q4. Find dy/dx: $(\cos x)^y = (\cos y)^x$. [2]

OR

Evaluate: $\int_{-1}^{2} |x^3 - x| dx$

Q5. Find the equation to the tangent at (0, 0) on the curve $y = 4x^2 - 2x^3$ [2]

Q6. Evaluate: $\int_0^{\pi/4} \log(1 + tanx)\, dx$ [4]

Q7. Show that $\sin^{-1}(5/13) + \cos^{-1}(3/5) = \tan^{-1}(63/16)$ [4]

Q8. Evaluate: $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x) dx$ [4]

Q9. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

OR

If $y = (tan^{-1}x)^2$, then show that: $(x^2 + 1)y_2 + 2x(x^2 + 1)y_1 = 2$. [4]

Q10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die? [4]

OR

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Q11. Three friends Rahul, Ravi and Rakesh went to a vegetable market to purchase vegetable. From a vegetable shop Rahul purchased 1 kg of each Potato, Onion and Brinjal for a total of Rs. 21. Ravi purchased 4 kg of each Potato, 3 kg Onion and 2 kg Brinjal for a total of Rs. 60 while Rakesh purchased 6 kg of each Potato, 2 kg Onion and 3 kg Brinjal for a total of Rs. 70.

(1) If the cost of Potato, Onion anD Brinjal are Rs. x, Rs. y, Rs. z per kg respectively, then convert above situation into system of linear equation.

(2) Convert the above system of linear equations in (1) in the form of AX = B.

(3) Find the cost of Potato, Onion and Brinjal. [5]

Q12. Find the solution of differential equation: $(x - y)\frac{dy}{dx} = x + 2y$

[5]

Q13. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum? [6]

Q.14. Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X. [6]

SECTION-B (15 Marks)

Q.15. [5× 1]

(1) If θ is the angle between two vectors a and b then |a.b|=|a×b| then θ =

(a) 0 (b) π/4 (c) π/2 (d) π.

(2) If a and b are two collinear vectors, then which of the following are incorrect:

(a) b = λa, for some scalar λ

(b) a = ±b

(c) the respective components of a and b are not proportional

(d) both the vectors a and b have same direction, but different magnitudes.

(3) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, find K such that $\vec{a} + K\vec{b}$ is perpendicular to $\vec{c}$ is

(a) 0 (b) 5 (c) 4 (d) 2

(4) Find | a | and | b |, if (a + b) · (a − b) = 8 and | a |= 8 | b |

(5) Find a vector of magnitude of 20 units and parallel to the vector $4\hat{i}+3\hat{j}-2\hat{k}$.

Q.16. i) Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line r = 2 iˆ - jˆ + 2 kˆ + l (3 iˆ + 4 jˆ + 2 kˆ) and the plane r .(iˆ - jˆ + kˆ) = 5 .

OR

ii) Find the projection of the vector iˆ + 3 ˆj + 7kˆ on the vector 7iˆ − ˆj + 8kˆ. [2]

Q17. Find the image and the foot of perpendicular from the point having position vector $\hat{\imath}+3\hat{\jmath}+4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{\imath}-\hat{\jmath}+\hat{k})+3=0$. [4]

OR

Find the vector equation of the plane passing through the intersection of the planes r .(2 iˆ + 2 jˆ - 3 kˆ) = 7r , r .(2 iˆ + 5 ˆj + 3 kˆ) = 9 r and through the point (2, 1, 3).

Q18 Find the area of the region bounded by the parabola $y=x^2$ and the line y=x+2 at x-axis. [4]

SECTION C (15 Marks)

Q19. Choose the correct option for each of the following questions.

(1)The total revenue in Rupees received from the sale of x units of a product is given by R(x) = $3x^2$ + 36x + 5. The marginal revenue, when x = 15 is

(a) 116 (b) 96 (c) 90 (d) 126 [1]

(2) The cost function of a firm is given by C(x) = $2x^2$ + x -5, the the marginal cost when x= 4 is

(a) 15 (b) 16 (c) 17 (d) 18 [1]

(3) The cost of manufacturing x units of a commodity is $27+15x+3x^2$. Find the output where $AC=MC$. [1]

Q20. Find the regression coefficients b_{yx} and b_{xy} when the line of regressions are $6x-5y+10=0, 10x-3y-28=0$. Also find the angle between them. [2]

Q21. Find the regression coefficient of x on y from the following data:

$\sum x=15, \sum y=15, \sum y^2=49, \sum xy=44, n=5$, Also find the value of x when $y=7$. [2]

Q22. The cost function given by $C(x)=\frac{1}{3}x^3-5x^2+30x-45$, where x is the number of unit produced. The selling price per unit is Rs. 6.

(1) Find the profit function $p(x)$.

(2) Find the interval where the profit is increasing.

(3) What number of articles should be sold to maximize profit? [4]

Q23. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture. [4]